Exercises Part 2
Logic

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Session 1

Exercise 0.1 *Show that* $s \oplus t$ *is equivalent to* $(p \wedge \neg q) \vee (\neg p \wedge q)$

Session 2

Exercise 0.2 *Give truth tables for the following expressions*

- $(a \Rightarrow b) \wedge (b \Rightarrow a)$
- $(a \Rightarrow b) \wedge (b \Rightarrow c)$

Have a crack at Problems 13, 14, and 15 in Stein *et al.*

Exercise 0.3 (Video «The universal quantifier») *Consider two statements*

- *every student passed the exam*
- *some student failed the exam*

Define predicate symbols and formulate the two statements in symbolic form.

What relationship exists between the statements? (Implication? Equivalence? Other?)

Session 4

Exercise 0.4 *Prove that if* m *is even and* n *is odd, then* $m + n$ *is odd.*

Exercise 0.5 *Give an example, in plain English (or Norwegian), of a statement which has the form* $\forall x \in U,\ \exists y \in V,\ p(x, y)$. *This may be a mathematical or an everyday statement; whatever you prefer.*

Using the same $p(x, y)$, *write the statement of the form* $\exists y \in V,\ \exists x \in U,\ p(x, y)$. *in plain English or Norwegian.*

Comment on whether 'exists' and 'for all' commute.

Exercise 0.6 *Recall the insertion sort algorithm from the first week:*

```
1   for i = 2 to n
2       j = i
3       while (j ≥ 2) and (A_j < A_{j-1})
4           exchange A_j and A_{j-1}
5           j = j-1
```

The number of exchanges required (in Line 4) may depend on the input list. So let $N(A)$ be the number of exchanges required for the list A. Using the counting argument you hopefully have already prooved that $N(A) \leq n(n-1)/2$.

- *Formulate in symbolic form, the claim that the bound cannot be improved, i.e. the full number of $n(n-1)/2$ exchanges is required for some list(s).*
- *Prove this claim.*

Exercise 0.7 *Compare*

$$(\exists x \in U,\ p(x)) \wedge (\exists y \in U,\ q(y)) \tag{1}$$

$$(\exists z \in U,\ p(z) \wedge q(z)). \tag{2}$$

Why are the two statements not equivalent?

Now, compare

$$(\exists x \in U,\ p(x)) \vee (\exists y \in U,\ q(y)) \tag{3}$$

$$(\exists z \in U,\ p(z) \vee q(z)). \tag{4}$$

Are these two expressions equivalent? Why?

Session 5

Exercise 0.8 *Prove that if $x^3 > 27$ then $x > 3$.*

Exercise 0.9 *Prove that $\sqrt{3}$ is irrational.*

Remember that any rational number r may be written as $r = n/m$ where n and m are integers ($m \neq 0$). If this is not possible, we say that r is irrational.

Exercise 0.10 *Prove that if m is an integer and m^2 is even, then m must be even.*

Exercise 0.11 *Rewrite the following argument in symbolic form, and decide whether or not it is a valid argument.*

1. *If you fail mathematics, the you will not get your engineering degree.*
2. *You get your engineering degree.*
3. *Therefore you did not fail the mathematics module.*