Partitioning and Pitfalls

Basic Counting

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Autumn 2013 – Part 1/Session 2/Video 1B
Recorded: 4th July 2014

Selection sort

Selection sort is a simple algorithm to sort all the elements in an array in increasing order.

Input Array `A` of length `n`

Output The same array `A` sorted in place.

```
1  for out_idx := 1 to n-1
2      for in_idx := out_idx+1 to n
3          if A[out_idx] > A[in_idx]
4              swap A[out_idx] with A[in_idx]
```

Question *How many comparisons (line 3) are made to sort the array?*

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The Sum Principle

Where can we go wrong

Assuming that

$$S = \bigcup_{i=1}^{n-1} S_i.$$

Under what conditions can we conclude that

$$|S| = \sum_{i=1}^{n-1} |S_i|?$$

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The Sum Principles

Condition

- Risk of double counting
- What if some element $x \in S_j$ and $x \in S_k$ with $j \neq k$.
- x is only one element in the union

$$S = \bigcup_{i=1}^{n-1} S_i.$$

- ... but x is counted twice in the sum

$$|S| = \sum_{i=1}^{n-1} ?$$

  - for $i = j$ and for $i = k$

The Sum Principle applies if and only if the constituent sets S_i are mutually disjoint.

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The Sum Principle

Definition #1

The size of a union of a family of mutually disjoint finite sets is the sum of the sizes of the sets.

Stein et al.

Exercise

Consider a 12-member club. In how many ways can they elect a chair and a secretary?