

Summing Consecutive Integers

Counting

Prof Hans Georg Schaathun

Høgskolen i Ålesund

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Selection sort

Input Array A of length n

Output The same array A sorted **in place**.

```
1  for out_idx := 1 to n-1
2      for in_idx := out_idx+1 to n
3          if A[out_idx] > A[in_idx]
4              swap A[out_idx] with A[in_idx]
```

- We have agreed that Line 3 is run

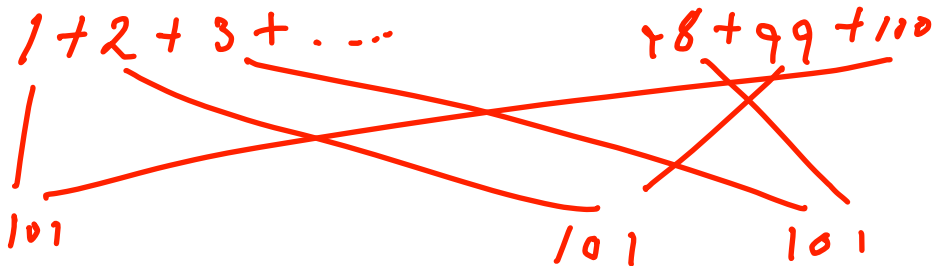
$$\sum_{i=1}^{n-1} (n-i) = \sum_{j=1}^{n-1} j$$

times

Can we find a neat closed-form expression for the sum?

Gauss' problem

Karl Friedrich Gauss (1777–1855)



$$101 \cdot 50 = \underline{\underline{5050}}$$

Gauss' problem

Karl Friedrich Gauss (1777–1855)

$$\begin{aligned} \sum_{j=1}^n j &= \sum_{j=1}^{n/2} \underbrace{(j + (n - j))}_{n+1} \quad n = 100 \\ &= \sum_{j=1}^{n/2} n+1 = \frac{n}{2}(n+1) \\ &= \frac{n \cdot (n+1)}{2} \end{aligned}$$

101

Even n

$$\sum_{j=1}^n j$$

$$\left. \begin{array}{c} 1 \\ 2 \\ 3 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \underline{\frac{n}{2}} \end{array} \right\} \frac{n}{2}$$

$$\begin{array}{l} n = n+1 \\ n-1 = n+1 \\ n-2 = n+1 \\ \vdots \\ \vdots \\ \vdots \\ \frac{n}{2} + 1 = n+1 \end{array}$$

$$\frac{n}{2} \cdot (n+1)$$

Conclusion

$$\sum_{j=1}^n j = \frac{n \cdot (n+1)}{2}$$