Summing Consecutive Integers

Counting

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Selection sort

Input Array A of length n

Output The same array A sorted in place.

```
1  for out_idx := 1 to n−1
2      for in_idx := out_idx+1 to n
3          if A[out_idx] > A[in_idx]
4              swap A[out_idx] with A[in_idx]
```

- We have agreed that Line 3 is run

$$\boxed{\sum_{i=1}^{n-1}(n-i)} = \sum_{j=1}^{n-1} j$$

times

Can we find a neat closed-form expression for the sum?

Gauss' problem

Karl Friedrich Gauss (1777–1855)

$$1 + 2 + 3 + \dots + 98 + 99 + 100$$

$101 \qquad 101 \qquad 101$

$$101 \cdot 50 = \underline{5050}$$

Gauss' problem

Karl Friedrich Gauss (1777–1855)

$$\sum_{j=1}^{n} j = \sum_{j=1}^{n/2} \underbrace{(j + (n-j))}_{n+1} \qquad n = 100$$

$$101$$

$$= \sum_{j=1}^{n/2} n+1 = \frac{n}{2}(n+1)$$

$$\boxed{= \frac{n \cdot (n+1)}{2}}$$

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Even n

$$\sum_{j=1}^{n} j$$

$$\frac{n}{2}\left\{\begin{array}{lll} 1 & n & = n+1 \\ 2 & n-1 & = n+1 \\ 3 & n-2 & = n+1 \\ \vdots & \vdots & \\ \frac{n}{2} & \frac{n}{2}+1 & = n+1 \end{array}\right.$$

$$\frac{n}{2}\cdot(n+1)$$

Conclusion

$$\boxed{\sum_{j=1}^{n} j = \frac{n\cdot(n+1)}{2}}$$