Counting dinner combinations
Solution example

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Exercise 1 *A dinner meal ought to comprise both starch and protein. Suppose you have the options of potatoes, rice, and spaghetti for the starch and beef, chicken, or meatballs for the protein.*

How many different dinners can you cook? Assume that you are allowed only one ingredient of each type.

1 Solution

We will solve this exercise using the pattern from the video lectures. The first step is to take the concrete (practical) problem and put it in a mathematical (formal) form.

1.1 Step 1: Formalisation

A dinner is a pair (x, y), where

$$x \in A = \{\text{beef}, \text{chicken}, \text{meatball}\} \tag{1}$$

$$y \in B = \{\text{spaghetti}, \text{rice}, \text{potato}\}. \tag{2}$$

For the sake of brevity, we assign symbols to the ingredients, and write $A = \{b, c, m\}$ and $B = \{s, r, p\}$.

Now, the set of possible dinners can be written as

$$D = \{(x, y) \mid x \in A, y \in B\} = A \times B.$$

We need to find $|D|$

1.2 Step 2: Partitioning

We partition D into subsets D_x with the protein x fixed. In other words, we write

$$D = \bigcup_{x \in A} D_x, \quad \text{where} \tag{3}$$

$$D_x = \{(x, y) \mid y \in B\}. \tag{4}$$

Since the dinners from different subsets D_x have different proteins, the subsets must be pairwise disjoint. Hence they form a partitioning.

1.3 Step 3: Counting

It is easy to see that D_x has one element for each element of B. Hence

$$|D_x| = |B| = 3. \tag{5}$$

It is also easy to see that there are 3 partitions D_x, one for each element of A.

We can use the product principle to see that

$$|D| = |A| \cdot |B| = 3 \cdot 3 = 9,$$

which is the required answer.