Partitioning

Using the Sum Principle

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Selection sort

Recall selection sort

Input Array A of length n

Output The same array A sorted in place.

```
1  for out_idx := 1 to n−1
2      for in_idx := out_idx+1 to n
3          if A[out_idx] > A[in_idx]
4              swap A[out_idx] with A[in_idx]
```

- Let S be the set of executions of Line 3.
- We calculated $|S|$ by considering sets $S_{\texttt{out\_idx}}$
    - executions during a given iteration of the outer loop.

$$S = \bigcup_{i=1}^{n-1} S_i$$

Partitioning

We used a trick known as partitioning.

- The set S is partioned into sets $S_1, S_2, \ldots$
- Partitioning requires mutually disjoint parts S_i
- If we included x in S_i, we must not include it in any other S_j ($j \neq i$).

Definition (Partitioning)

A family of sets $\{S_1, S_2, \ldots S_k\}$ is a partitioning of S if and only if

1. $S = \bigcup_{i=1}^{n-1} S_i$
2. $S_i \cap S_j = \emptyset$ whenever $i \neq j$.

The Sum Principle

Definition #2

If a finite set S has been partitioned into blocks, then the size of S is the sum of sizes of the blocks.

Stein et al.

Abstraction

1. We started with a practial problem
   - counting operations in an algorithm
2. Translated the problem into mathematics (set theory)
3. Used generic theory and principles to solve it

Exercise of abstraction

- The mathematical solution is abstract, and independent of the actual problem
- Yet it applies to the concrete problem
- Powerful reuse of theory and insights