The Product Principle

Using partitioning on lists and permutations

Prof Hans Georg Schaathun

Høgskolen i Ålesund

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The Product Principle

- We have seen the product principle twice
  - in different contexts
- Partition a set $S = \cup_{i=1}^{m} S_i$
  - disjoint S_i
  - same size $|S_i| = n$
  - Product Principle
    - $|S| = m \cdot n$
- A list of of k elements from an n-set T
  - Choose k times
  - n options each time
  - Product principle:
    - $n \cdot n \cdot n \cdot \ldots \cdot n = n^k$ possible lists

Partitioning the set of lists

Two element lists

How many lists of two elements exist on the set T of n elements?

- Let's use *partitioning*
- Set S of possible lists
  - $S = \{(x, y) : x, y \in T\}$
- Partitioning $S = \cup_i S_i$
- Let $S_i = \{(i, y) : y \in T\}$
  - The S_i are disjoint
  - $S = \cup_{i \in T} S_i$
- Product principle $|S| = n^2$

Partitioning sets of longer lists

How many lists of three elements exist on the set T of n elements?

- Set S' of possible lists
  - $S' = \{(x, y, z) : x, y.z \in T\}$
- Partitioning $S' = \cup_i S'_i$
- Let $S'_i = \{(i, y, z) : y, z \in T\}$
  - The S'_i are disjoint
  - $S' = \cup_{i \in T} S'_i$
  - $S'_i \cong S$ (set of two-element lists)
- Product principle $|S| = n \cdot n^2$

one can continue recursively for k-element lists

Exercise

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Consider two sets M and N of size m and n respectively. How many ordered pairs (x, y) exist, so that $x \in M$ and $y \in N$?

HØGSKOLEN I ÅLESUND
Aalesund University College