Relations and Functions

Relations

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Høgskolen i Ålesund

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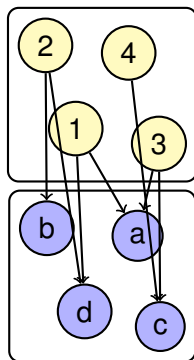
Recall: Relations

Definition

A relation from X to Y is a set R of ordered pairs (x, y) where $x \in X$ and $y \in Y$.
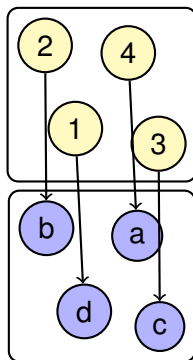
For instance,

- $a = b$
- $a < b$ (a relation on any ordered set)
- $a|b$ (a is divisible by b)
- $A \subset B$ (for subsets of some universe)
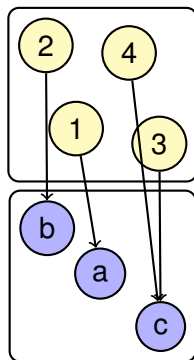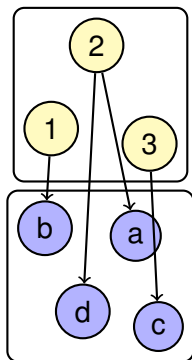
Four types of relations

Four types of relations

Consider a relation $R \subset X \times Y$ from X to Y.

Many-to-many where there are no restrictions. An element may be related to an arbitrary number of elements in other set.

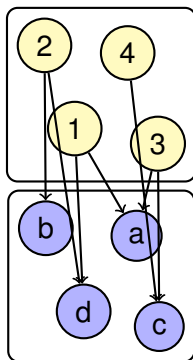Many-to-one For any $x \in X$ there is a unique $y \in Y$ such that xRy

One-to-many For any $y \in Y$ there is a unique $x \in X$ such that xRy
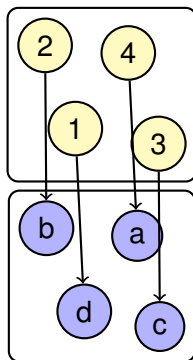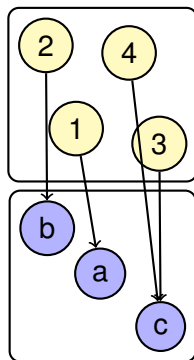
One-to-one Both one-to-many and many-to-one.

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Functions as relations
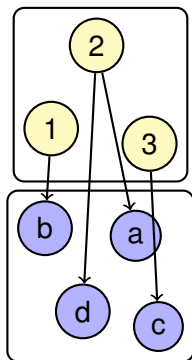

Bijection

Many-to-many One-to-one Many-to-one One-to-many
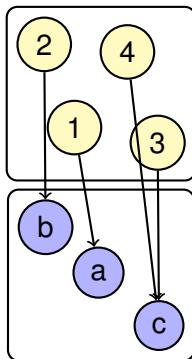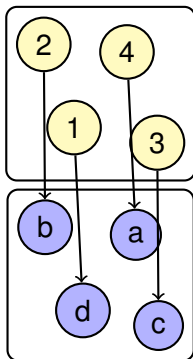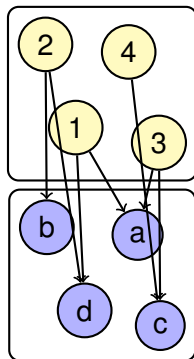
Functions as relations

Functions as relations
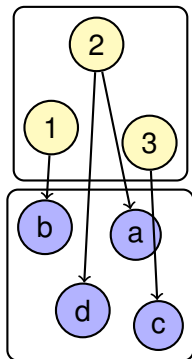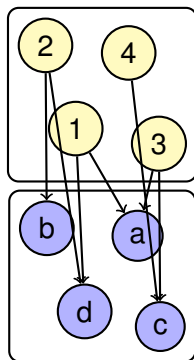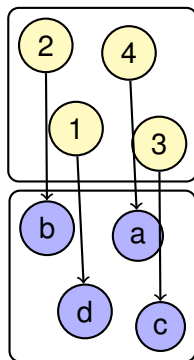
Functions as relations

Not a function
Many-to-many

Bijection
One-to-one

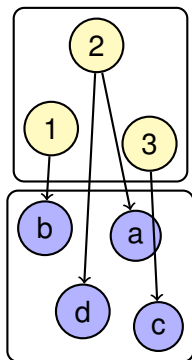Function $X \rightarrow Y$
Many-to-one
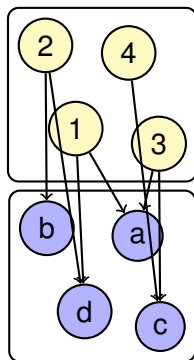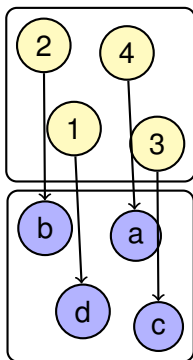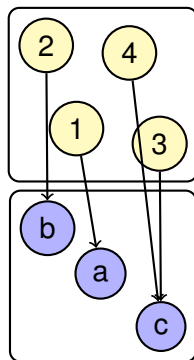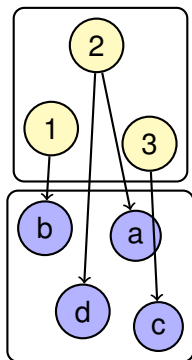
$Y \rightarrow X$
One-to-many

Functions are relations

Function $f : X \to Y$

$$f(x) = y$$

Relation $R \subset X \times Y$

$$(x, y) = (x, f(x)) \in R$$

Every function is a relation, either one-to-many or one-to-one.

- Many-to-many and many-to-one relations are not functions $X \to Y$.
- A many-to-one relation is a function $g : Y \to X$.

Exercise

1. Consider the following relations and label them as many-to-many, one-to-many, many-to-one, or one-to-one.
   1. *x* is a friend of *y*
   2. *x* is the son of *y*
   3. *x* is the mother of *y*
   4. *x* is married to *y*
   5. *x* lives at *y* (address)
2. Which of the above relations can be cast as functions $x \mapsto y$?