If and only if Propositional calculus

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if and only if

- Implication: s ⇒ t expressed in different ways
 - t only if s
 - s if t
- What does s if and only if t mean?
 - $(s \text{ if } t) \land (s \text{ only if } t)$ • $(s \Leftarrow t) \land (s \Rightarrow t)$
- if and only if is a two-way implication
 - $s \Leftrightarrow t$

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Ambiguous if

If you get an A in mathematics, then we go out to dine at XL Diner.

some parents

- Suppose you get a B,
- and yet you all go out to dine at XL.

Was the original promise true or false?

The logicians answer

s :=you get an A in mathematics t :=we go out to dine at XL Diner

s	t	$s \Rightarrow t$
Т	Τ	Т
Τ	F	F
F	Τ	Т
F	F	Т

Principle (Excluded middle)

A statement is true exactly when it is not false.

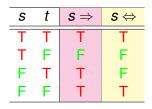
The other answer

We sometimes understand if and only if, even when just if was said.

- t states a reward for making s true
- It would be bad paedagogy to make t true when s is fales

But: that's more than if says.

Implication versus equivalence



Exercise

- Give an example in plain English or Norwegian where «if» appears to mean «if and only if» (or where you think it would for many people).
- Similarly, give an example where you don't think «if» would mean «if and only if».