

If and only if

Propositional calculus

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if and only if

- Implication: $s \Rightarrow t$ expressed in different ways
 - t only if s
 - s if t
- What does s if and only if t mean?
 - $(s \text{ if } t) \wedge (s \text{ only if } t)$
 - $(s \Leftarrow t) \wedge (s \Rightarrow t)$
- *if and only if* is a two-way implication
 - $s \Leftrightarrow t$

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Ambiguous *if*

If you get an A in mathematics, then we go out to dine at XL Diner.

some parents

- Suppose you get a B,
- and yet you all go out to dine at XL.

Was the original promise true or false?

The logicians answer

s := you get an A in mathematics

t := we go out to dine at XL Diner

s	t	$s \Rightarrow t$
T	T	T
T	F	F
F	T	T
F	F	T

Principle (Excluded middle)

A statement is true exactly when it is not false.

The other answer

*We sometimes understand **if and only if**, even when just **if** was said.*

- t states a reward for making s true
- It would be bad paedagogy to make t true when s is false

*But: that's more than **if** says.*

Implication versus equivalence

s	t	$s \Rightarrow$	$s \Leftrightarrow$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	T

Exercise

- 1 Give an example in plain English or Norwegian where «if» appears to mean «if and only if» (or where you think it would for many people).
- 2 Similarly, give an example where you don't think «if» would mean «if and only if».