Implication

Propositional calculus

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Propositional logic

- So far
  - making statements – predicates
  - combining statements with $\wedge$, $\vee$, $\oplus$, and $\neg$
- To be discussed
  - Arguments, deduction
  - Conditionals $\rightarrow$ if . . . then

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Implication

Many expressions to mean the same thing ...

- If s then t
- s implies t
- t follows from s
- t if s
- s only if t

Many words to express implication.

- Symbolic form: $s \Rightarrow t$ $(t \Leftarrow s)$
- or sometimes $s \rightarrow t$ $(t \leftarrow s)$

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For example

- Everyday
  - If Alice is ill, then she will not come to class
- Programming
  - if L.empty() then print(L.next())

For example, mathematics

If p is a prime, then $a^{p-1} \mod p = 1$ for each integer $a = 1, \ldots, p-1$

Fermat's little theorem

p is prime $\Rightarrow a^{p-1} \mod p = 1$ for each integer $a = 1, \ldots, p-1$

The ambiguity and variety of English terms is resolved by symbolic notation.

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Definitions

$$s \Longrightarrow t$$

hypothesis s is a called a hypothesis.
We say nothing about its truth value.

conclusion t is a called the conclusion.
We can conclude the truth values under certain conditions (s).

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Exercise

Show that the expression $s \Rightarrow t$ is equivalent to $\neg s \vee (s \wedge t)$.

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