Modus Ponens

Direct Proof

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Example

1. If the sun is shining, then we go swimming
2. the sun is shining
3. Therefore, we go swimming

$$s := \text{the sun is shining,} \tag{1}$$
$$t := \text{we go swimming} \tag{2}$$

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Principle (Modus Ponens)

From p and $p \Rightarrow q$, we can conclude q.

p	$p \Rightarrow q$	q
T	T	T
T	F	F
F	T	T
F	T	F

A proof from last session

$$\forall x \in U_2, (q(x) \Rightarrow p(x)) \text{ implies } \forall x \in U_1, p(x)$$

- Consider $x \in U_1$
- We need to prove $p(x)$
- $q(x)$ (by the definition of U_1)
- Now $x \in U_2$ (by definition of U_1)
- Hence $q(x) \Rightarrow p(x)$ (by left hand side)
- Hence $p(x)$, quod erat demonstrandum

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Summary

- A few patterns for valid arguments
- Modus ponents inference is a fundamental one
  - based on the tautology

$$(p \wedge (p \Rightarrow q)) \Rightarrow q$$

- Symbolic notation, tend to make the pattern clear
  - simple tool to validate an argument