

# Conditional proof

## Direct Proof

Prof Hans Georg Schaathun

Høgskolen i Ålesund

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# An example to be proved

*The sum of even integers is even.*

- Let  $p(x) := x$  is even
- Our claim can be written as

$$(p(y_1) \wedge p(y_2)) \Rightarrow p(y_1 + y_2)$$

# Conditional proof

## Principle (Conditional proof)

*If, by assuming just that  $p$  is true, we can prove  $q$ , then we can conclude that  $p \Rightarrow q$ .*

$$(p(y_1) \wedge p(y_2)) \Rightarrow p(y_1 + y_2)$$

- Lets assume  $p(y_1)$  and  $p(y_2)$ 
  - write  $y_1 = 2z_1$  and  $y_2 = 2z_2$
  - $z_1, z_2 \in \mathbb{Z}$
- Thus  $y_1 + y_2 = 2z_1 + 2z_2 = 2(z_1 + z_2)$ 
  - where  $z_1 + z_2$  is an integer
- Hence  $2(z_1 + z_2)$  is even and  $p(y_1 + y_2)$  is true.

