Revision Exercises — Set 1
Counting

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1 Some problems with solutions

Problem 1.1 *Consider a 12-member club.*

1. *In how many ways can they elect a chair and a secretary, assuming that no individual may hold both posts?*

> **Solution:** Let K be the club (set of club members). The set of possible leader/secretary combinations, can be written as
>
> $$S = \{(l, s) \mid l, s \in K, l \neq s\},$$
>
> i.e. the set of ordered pairs from K.
>
> We can partition $S = \cup S_l$ where S_l is the set of pairs with l as the leader. With $|K| = 12$ options for leader, we have 12 partitions, and $|S_l| = 11$ elements per partition. The product principle gives $|S| = 12 \cdot 11 = 132$.

2. *In how many ways can they elect two members for a steering committee?*

> **Solution:** A steering committee is a subset of the club. The number of k-element subsets of an n-element set is given by the binomial coefficient
>
> $$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$
>
> For a two-member committee, we have $\binom{12}{2} = 66$ choices.

3. *In how many ways can they elect a chair and two vice-chairs, assuming that no individual may hold more than one post?*

Solution: This problem can be solved with a partitioning as in 1-1. We have one partition S_l for each possible leader l, i.e. 12 partitions. Each partition has one element for each possible two-sets chosen from the remaining 11 members, forming the possible combinations of two vice-presidents. Thus $|S_l| = \binom{11}{2} = 55$, and the product principle gives $|S| = 12 \cdot 55 = 660$.

Problem 1.2 *Alice, Bob, Charlie and Denise organise a private table tennis tournament. They make it a round-robin tournament where each player plays each of the others exactly once.*

1. *Systematically write down all the matches.*

Solution:

Alice-Bob		
Alice-Charlie	Bob-Charlie	
Alice-Denise	Bob-Denise	Charlie-Denise

2. *Using your list (table) of matches, show how you use the Sum Principle to count the number of matches required.*

Solution: We have partitioned the matches into three disjoint groups, with 3, 2, and 1 matches. The total number of matches is the sum of the number per group, i.e. $3 + 2 + 1 = 6$.

3. *How can you specify the number of matches using a binomial coefficient? Why is a binomial coefficient appropriate?*

Solution: A match is a subset of two elements from the set of four players. The number of ways to choose such set is $\binom{4}{2}$.

2 Session 1

Exercise 2.1 *There are 19 male and 132 female nursing students on a degree course.*

1. *In how many ways can you select two student representatives such that both genders are represented?*
2. *In how many ways can you select a single student representatives for the course?*
3. *What counting principles do you use to answer 1 and 2?*

Exercise 2.2 *You roll two dice. (Normal, six-sided dice.)*

1. *How many combinations of dots are possible on the two dice, assuming that you cannot tell the two dice apart?*
2. *Suppose the two dice have different colour. How many combinations of dots are possible now?*
3. *Consider the sum of dots. How many combinations give the same sum? Give your answer for each possible value of the sum.*
4. *Which is the most likely sum to get? Explain why?*
5. *How much more likely are you to get the most likely sum, compared to the least likely sum?*

3 Session 2

Exercise 3.1 *Consider a deck of 52 cards and five players. Each player is dealt a single card. How many possible deals exist?*

Exercise 3.2 *Consider a swimming race of eight contestants. How many combinations of medalists (gold, silver, bronze) are possible? Give reasons for your answers?*

Exercise 3.3 (Rosen p. 402, Problem 7) *How many bit strings of 10 bits contain*

- *exactly four ones?*
- *at most four ones?*
- *at least four ones?*
- *an equal number of zeros and ones?*

Give reasons for your answers?

Exercise 3.4 *Consider an arbitrary 10-bit bit string $\vec{s}$. How many 10-bit strings exist that*

- *differ from $\vec{s}$ in exactly four places?*
- *differ from $\vec{s}$ in at most four places?*

Give reasons for your answers?

Exercise 3.5 *Consider a 9-element set S. How many subsets of S exist with an odd number of elements? Give reasons for your answer?*

Problem 3.1 *For security reasons, we often want to make the password space (set of valid passwords) as large as possible.*

Still considering passwords of four to eight characters, how much larger does the password space become if we allow digits as well as the 52 upper and lower case letters?

Give the answer as a factor. E.g. the new password space is x times larger than the old one.