Revision Exercises Week 2
Counting

Hans Georg Schaathun

12th November 2015

Problem 0.1 *Consider the relation $\sim$ relating x to y if $x^2 = y^2$.*

1. *Show that $\sim$ is a reflexive relation.*
2. *Show that $\sim$ is a symmetric relation.*
3. *Show that $\sim$ is a transitive relation.*
4. *What do we mean when we say that $\sim$ is an equivalence relation?*
5. *Describe the equivalence classes of $\sim$.*

Exercise 0.1 *Calculate the following*

1. $\binom{7}{3}$
2. $\binom{9}{4}$
3. $\binom{14}{4}$
4. $\binom{14}{10}$.
5. $\binom{620}{1}$
6. $\binom{620}{619}$
7. $\binom{620}{618}$

Exercise 0.2 *Give two proofs that*

$$\binom{n}{k} = \binom{n}{n-k}$$

Exercise 0.3 (Freely from Stein *et al* 1.1 Exercise 9) *Using the formula for $\binom{n}{2}$, it is easy to see that*

$$n\binom{n-1}{2} = \binom{n}{2}(n-2)$$

Find an intuitive and conceptual argument that this equation holds, using the fact that $\binom{n}{2}$ represents the number of two-element subsets.

Hint! *You may think in terms officers and committees in a club, as in Exercise* **??** *Question 3.*

Exercise 0.4 (Stein *et al* 1.3 Exercise 8) *Consider a Cartesian coordinate system with integer coordinates. How many different paths exist from the origin $(0, 0)$ to the point (m, n) where each path is built from m horizontal and n vertical line segments, each of length 1?*

Exercise 0.5 (Stein *et al* 1.3 Exercise 18) *Apply calculus and the binomial theorem to $(1 + x)^n$ to show that*

$$\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \ldots = n2^{n-1}$$