Revision Exercises Week 2
Counting

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Problem 0.1 *Consider the relation $\sim$ relating x to y if $x^2 = y^2$.*

1. *Show that $\sim$ is a reflexive relation.*
2. *Show that $\sim$ is a symmetric relation.*
3. *Show that $\sim$ is a transitive relation.*

> **Solution:** We verify the three properties as follows.
>
> **Reflexivity** $x^2 = x^2$ and hence $x \sim x$
>
> **Reflexivity** if $x^2 = y^2$ then $y^2 = x^2$, and hence $y \sim x$ whenever $x \sim y$
>
> **Transitivity** Suppose $x \sim y$ and $y \sim z$. This gives $x^2 = y^2$ and $y^2 = z^2$, and it follows that $x^2 = z^2$ and $x \sim z$ as required.

4. *What do we mean when we say that $\sim$ is an equivalence relation?*

> **Solution:** A relation is an equivalence if it is transitive, symmetric, and reflexive.

5. *Describe the equivalence classes of $\sim$.*

> **Solution:** The equivalence class $[y]$ of y contains the elements solving $x^2 = y^2$, which is $\pm y$ (assuming real (not complex) numbers). Each equivalence class contains the two elements $\pm y$ for some y, except for one class which only contains a single element, 0.

Exercise 0.1 *Calculate the following*

1. $\binom{7}{3}$
2. $\binom{9}{4}$

3. $\binom{14}{4}$
4. $\binom{14}{10}$.
5. $\binom{620}{1}$
6. $\binom{620}{619}$
7. $\binom{620}{618}$

Exercise 0.2 *Give two proofs that*

$$\binom{n}{k} = \binom{n}{n-k}$$

Exercise 0.3 (Freely from Stein *et al* 1.1 Exercise 9) *Using the formula for* $\binom{n}{2}$*, it is easy to see that*

$$n\binom{n-1}{2} = \binom{n}{2}(n-2)$$

Find an intuitive and conceptual argument that this equation holds, using the fact that $\binom{n}{2}$ *represents the number of two-element subsets.*

Hint! *You may think in terms officers and committees in a club, as in Exercise* **??** *Question 3.*

Exercise 0.4 (Stein *et al* 1.3 Exercise 8) *Consider a Cartesian coordinate system with integer coordinates. How many different paths exist from the origin* $(0,0)$ *to the point* (m,n) *where each path is built from* m *horizontal and* n *vertical line segments, each of length 1?*

Exercise 0.5 (Stein *et al* 1.3 Exercise 18) *Apply calculus and the binomial theorem to* $(1+x)^n$ *to show that*

$$\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \ldots = n2^{n-1}$$