Haust 2016 oppg. 1 d)

$$\sigma^2 = \int_{-\infty}^{\infty} f(x)(x-\mu)^2\,dx = \int_0^1 \tfrac{1}{2}x(x-\mu)^2\,dx + \int_1^2 \tfrac{1}{2}(x-\mu)^2\,dx + \int_2^3 \left(\tfrac{3}{2}-\tfrac{1}{2}x\right)(x-\mu)^2\,dx$$

$$\mu = 1\tfrac{1}{2}$$

$$\int_0^1 \tfrac{1}{2}x(x^2-3x+2{,}25)\,dx = \int_0^1 \tfrac{1}{2}x^3 - 1\tfrac{1}{2}x^2 + 1\tfrac{1}{8}x\,dx$$

$$= \left[\tfrac{1}{8}x^4 - \tfrac{1}{2}x^3 + \tfrac{9}{16}x^2\right]_0^1 = \tfrac{1}{8} - \tfrac{1}{2} + \tfrac{9}{16} = \frac{2-8+9}{16} = \underline{\tfrac{3}{16}}$$

$$\int_2^3 \left(\tfrac{3}{2}-\tfrac{1}{2}x\right)(x^2-3x+2{,}25)\,dx = \int_2^3 -\tfrac{1}{2}x^3 + \left(\tfrac{3}{2}+\tfrac{3}{2}\right)x^2 - \left(\tfrac{9}{2}+1\tfrac{1}{8}\right)x + \tfrac{3}{2}2{,}25\,dx$$

$$= \int_2^3 -\tfrac{1}{2}x^3 + 3x^2 - 5\tfrac{5}{8}x + 3\tfrac{3}{8}\,dx = \cancel{\left[-\tfrac{27}{2} + 27\right.}$$

$$= \left[-\tfrac{1}{8}x^4 + x^3 - \frac{5\frac{5}{8}}{2}x^2 + 3\tfrac{3}{8}x\right]_2^3 = -\tfrac{1}{8}(81-16) + (27-8) - \frac{5\frac{5}{8}}{2}(9-4) + 3\tfrac{3}{8}$$

$$= -\tfrac{1}{8}\cdot 65 + 19 - \frac{5\cdot\frac{5}{8}}{2}\cdot 5 + 3\tfrac{3}{8} = -8\tfrac{1}{8} + 19 - \tfrac{25}{2} - \tfrac{25}{16} + 3\tfrac{3}{8}$$

$$= 11 + 3\tfrac{2}{8} - 12\tfrac{1}{2} - 1\tfrac{9}{16} = 1 + \tfrac{2}{8} - \tfrac{1}{2} - \tfrac{9}{16} = \cancel{\tfrac{1}{2} + \tfrac{1}{4} - \tfrac{1}{2} - \tfrac{1}{16} = \tfrac{1}{4} - \tfrac{1}{16}} = \tfrac{3}{16}$$

$$= 1 + \frac{4-8-9}{16} = \underline{\tfrac{3}{16}}$$

$$\int_1^2 \frac{1}{2}(x-\mu)^2\,dx = \int_1^2 \frac{1}{2}(x^2 - 3x + 2{,}25)\,dx$$

$$= \left[\frac{1}{6}x^3 - \frac{3}{4}x^2 + 1\frac{1}{8}x\right]_1^2$$

$$= \frac{1}{6}[8-1] - \frac{3}{4}[4-1] + 1\frac{1}{8}$$

$$= 7/6 - 9/4 + 1\frac{1}{8} = 1\frac{1}{2} - 2\frac{1}{4} + 1\frac{1}{8}$$

$$= \frac{4}{8} - \frac{2}{8} + \frac{1}{8} = \frac{3}{8}$$

$$\int_0^3 f(x)(\mu-x)^2\,dx = \frac{3}{16} + \frac{3}{8} + \frac{3}{16} = \frac{6}{8} = \boxed{\frac{3}{4}}$$