Decoding Error Probabilities
A Case for Estimation

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7th January 2014

Decoding Error

- Sending a bit on BSC is a Bernoulli trial.
  - either correct transmission (Success) or bit error (Failure).
- Consider sending a k-bit message word **m**
  - channel with error control
  - Decoder gives an estimate **m̂**
  - We get either
    1. correct decoding (Success)
    2. decoding error (Failure)
- This is also a Bernoulli trial
  - Success probability p
  - or decoding error probability $p_e = 1 - p$.

p_e

World of Things

World of Forms

Pe

Concrete	Abstract
Experiment	Theory
Observed values	Probability distribution
Stochastic variables	Unknown parameters
Estimate	Unknown value
Things	Ideas

estimate

A Monte Carlo Experiment

$\Rightarrow$ n Bernoulli trials

- Recall the Monte Carlo simulator.
  1. Test the system n times
  2. Record the number X of errors (failures)

Exercise

What is the probability distribution of X?

$$X \sim B(n, p_e)$$

$$\frac{X}{n} \sim \text{ERROR RATE}$$

Desired values	Observable values
p_e	n, X

$$X \sim B(n, \boxed{p_e})$$

$$E(X) = n \cdot p_e$$

$$\boxed{E(X/n) = p_e}$$

$$\frac{X}{n} \approx p_e \quad \text{w/ high prob}$$

Point Estimator
Summary

- Error rate: $R = X/n$ — a stochastic variable
- Error probability: $p_e = E(R)$ — unknown parameter
- Observing enough instances of R X
  - we can make an opinion of the approximate value of $E(R) = p_e$
- $R \approx p_e$ with high probability
  - R is an estimator of p_e

Note the estimator is a stochastic variable.