# The Central Limit Theorem

## Probability Distributions of Estimators

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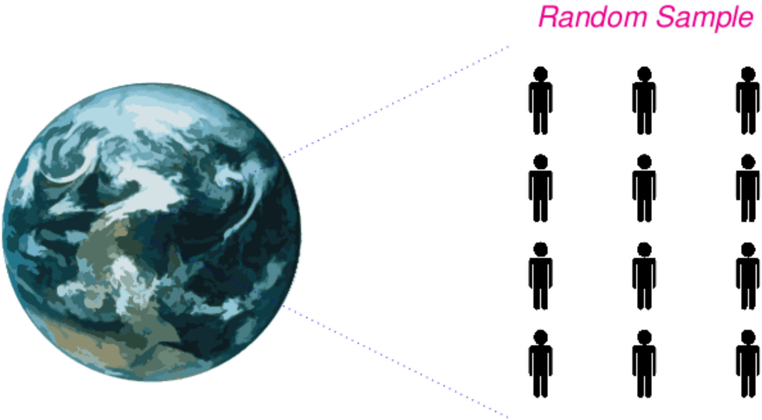
# Sample Mean




$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

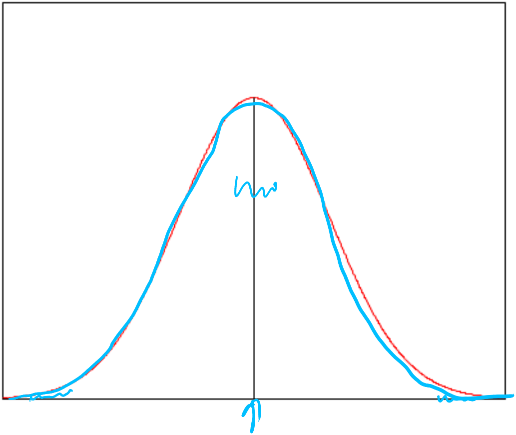
$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

# Central Limit Theorem

CLT

- Sample mean $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$
  - sum of independent variables
- Central Limit Theorem
  - any sum $X = \boxed{X_1 + X_2 + \ldots + X_n}$
  - of identically distributed independent variables $X_i$
  - regardless of the exact distribution of $X_i$
  - as $n \to \infty$, $X$ has always the same distribution
- This distribution, at the limit at infinity, is known as
  - the normal distribution, or
  - the Gaussian distribution

# The Gauss Curve



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

# The standard normal distribution

NORMALISATION

$N(0,1)$

1. $Y \sim N(\mu, \sigma)$
2. $Z = \frac{Y-\mu}{\sigma}$
3. $Z \sim N(0, 1)$

$$\boxed{Z = \frac{Y-\mu}{\sigma}}$$

$\mu = 0$
$\sigma = 1$

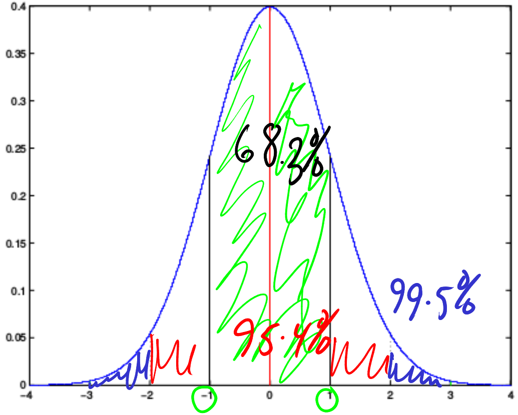
**Definition**

The Standard Normal Distribution is the Normal Distribution with $\mu = 0$ and $\sigma = 1$.

- We customarily normalise variables to use the Standard Normal Distribution
- E.g. for probability tables.

# The Gauss Curve

## The PDF of the standard normal distribution




$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\boxed{\begin{aligned} \mu &= 0 \\ \sigma &= 1 \end{aligned}}$$

# Summary

- Let $X = \boxed{X_1 + X_2 + \ldots + X_n}$
  - sum of $n$ identically distributed variables $X_i$
- When $n \to \infty$, $X \sim N(\mu, \sigma)$ — $X$ is normally distributed

## Exercise

*Find the following probabilities using the* `cdf()` *function in Matlab (or some other method):*

1. $P(Z \leq -2)$ *when* $Z \sim N(0,1)$
2. $P(Z \leq -1)$ *when* $Z \sim N(0,1)$
3. $P(Z \geq 1)$ *when* $Z \sim N(0,1)$
4. $P(-1 \leq Z \leq 1)$ *when* $Z \sim N(0,1)$

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