The Central Limit Theorem

Probability Distributions of Estimators

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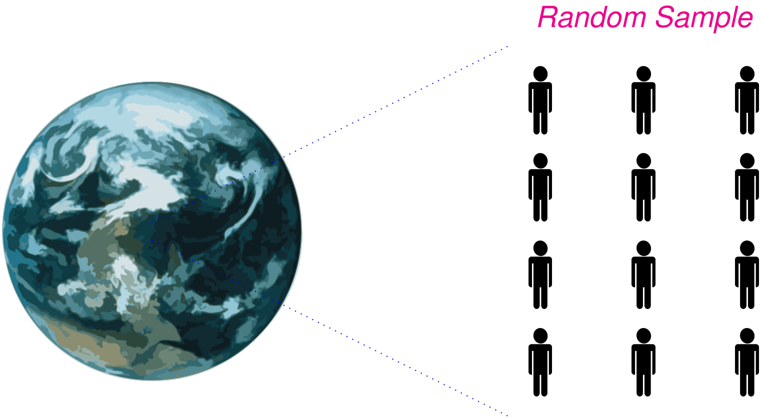
Sample Mean



$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$

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Central Limit Theorem

- Sample mean $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$
  - sum of independent variables
- Central Limit Theorem
  - any sum $X = X_1 + X_2 + \ldots + X_n$
  - of identically distributed independent variables X_i
  - regardless of the exact distribution of X_i
  - as $n \to \infty$, X has always the same distribution
- This distribution, at the limit at infinity, is known as
  - the normal distribution, or
  - the Gaussian distribution

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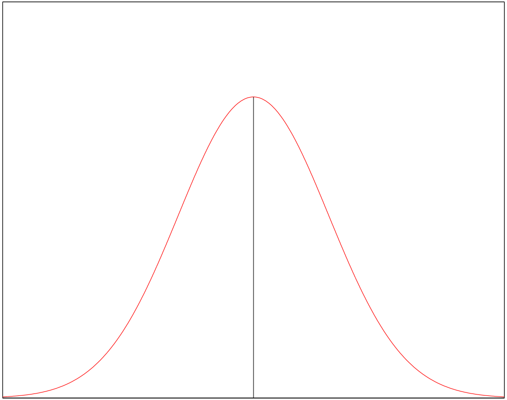
The Gauss Curve



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

The standard normal distribution

1. $Y \sim N(\mu, \sigma)$
2. $Z = \frac{Y-\mu}{\sigma}$
3. $Z \sim N(0, 1)$

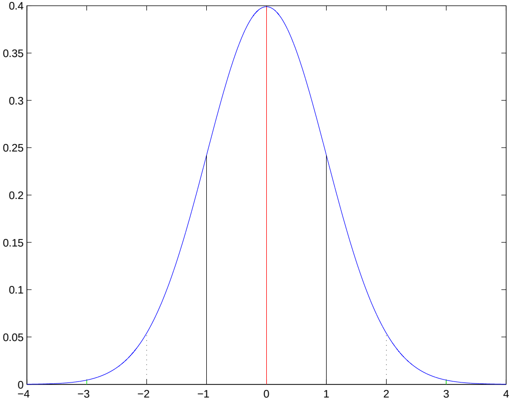
Definition

The Standard Normal Distribution is the Normal Distribution with $\mu = 0$ and $\sigma = 1$.

- We customarily normalise variables to use the Standard Normal Distribution
- E.g. for probability tables.

The Gauss Curve

The PDF of the standard normal distribution



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\mu = 0$$

$$\sigma = 1$$

Summary

- Let $X = X_1 + X_2 + \ldots + X_n$
  - sum of n identically distributed variables X_i
- When $n \to \infty$, $X \sim N(\mu, \sigma)$ — X is normally distributed

Exercise

Find the following probabilities using the `cdf()` function in Matlab (or some other method):

1. $P(Z \leq -2)$ *when* $Z \sim N(0, 1)$
2. $P(Z \leq -1)$ *when* $Z \sim N(0, 1)$
3. $P(Z \geq 1)$ *when* $Z \sim N(0, 1)$
4. $P(-1 \leq Z \leq 1)$ *when* $Z \sim N(0, 1)$

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