

Error Margin

Estimating the Mean with Known Variance

Prof Hans Georg Schaathun

Høgskolen i Ålesund

11th February 2014

Estimating the Mean

- What is the mean μ ?
 - Stochastic variable X
 - Known standard deviation σ
 - Unknown distribution
- The Sample Mean is a point estimator
 - n observations: x_1, x_2, \dots, x_n
 - Sample mean: $\hat{\mu} = \bar{x} = \sum x_i / n$
- How do we make a confidence interval?

Probability distribution

What is the probability distribution of \bar{X} ?

$$\bar{X} = \frac{X_1}{n} + \frac{X_2}{n} + \dots + \frac{X_n}{n}$$

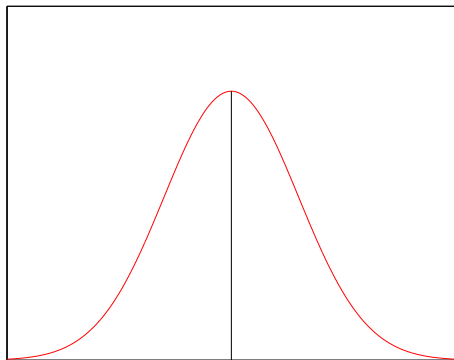
Variable	Expected value	Variance	Std.Dev.
X_i	μ	σ^2	σ
$\frac{X_i}{n}$	$\frac{\mu}{n}$	$\left(\frac{\sigma}{n}\right)^2$	$\frac{\sigma}{n}$
\bar{X}	$n \cdot \frac{\mu}{n}$ =	$n \cdot \left(\frac{\sigma}{n}\right)^2$ =	
\bar{X}	μ	$\frac{\sigma^2}{n}$	$\frac{\sigma}{\sqrt{n}}$

Central Limit Theorem: *For large n , \bar{X} has normal distribution.*

Estimation Error

- Estimation error $E = \bar{X} - \mu$
 - $E \sim N(0, \sigma/\sqrt{n})$
- Normalised $Z = \frac{E}{\sigma/\sqrt{n}} = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$
 - $Z \sim N(0, 1)$

Error Distribution



- $E = Z \cdot \sigma / \sqrt{n}$
- $P(Z \leq -z) = \alpha/2$
- $P(E \leq -e) = \alpha/2$

$\alpha/2$	z	e
2.5%	1.96	$1.96 \cdot \sigma / \sqrt{n}$
1.0%	2.33	$2.33 \cdot \sigma / \sqrt{n}$
0.5%	2.33	$2.58 \cdot \sigma / \sqrt{n}$

- We write $z_{\alpha/2}$

Matlab: $z = -\text{icdf}('norm', \alpha/2, 0, 1)$

Summary

- Error $E = \bar{X} - \mu$
- $z_{\alpha/2}$ is such that $P(Z \leq -z) = P(Z \geq z) = \alpha/2$
- $P(E \leq -z_{\alpha/2} \cdot \sigma/\sqrt{n}) = P(E \geq z_{\alpha/2} \cdot \sigma/\sqrt{n}) = \alpha/2$
 - $z = -\text{icdf}(\text{'norm'}, \alpha/2, 0, 1)$
- $P(|E| \geq z_{\alpha/2} \cdot \sigma/\sqrt{n}) = \alpha$
- With probability $\beta = 1 - \alpha$ (**confidence level**)
 - the error is within $\pm z_{\alpha/2} \cdot \sigma/\sqrt{n}$