Confidence Interval for the Mean

From Error Margin to Confidence Interval

Prof Hans Georg Schaathun

Høgskolen i Ålesund

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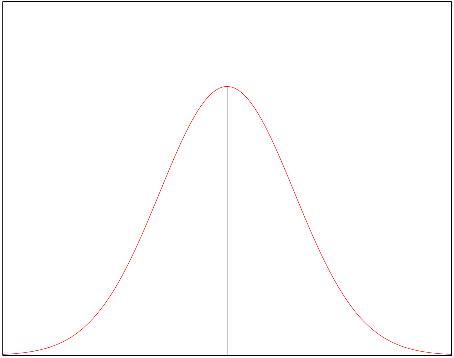
Error Margin



- $E = Z \cdot \sigma / \sqrt{n}$
- $P(Z \leq -z) = \alpha/2$
- $P(E \leq -e) = \alpha/2$

$\alpha/2$	z	e
2.5%	1.96	$1.96 \cdot \sigma/\sqrt{n}$
1.0%	2.33	$2.33 \cdot \sigma/\sqrt{n}$
0.5%	2.33	$2.58 \cdot \sigma/\sqrt{n}$

- We write $z_{\alpha/2}$

Matlab: `z = -icdf('norm', α/2, 0, 1)`

HØGSKOLEN I ÅLESUND
Aalesund University College

From Error Margin to confidence interval

- Probability $\beta = 1 - \alpha$:
  - $\mu - z_{\alpha_2} \cdot \sigma / \sqrt{n} \leq \bar{X} \leq \mu + z_{\alpha_2} \cdot \sigma / \sqrt{n}$
- Turn it around:
  - $\bar{X} - z_{\alpha_2} \cdot \sigma / \sqrt{n} \leq \mu \leq \bar{X} + z_{\alpha_2} \cdot \sigma / \sqrt{n}$
- Probability is still β
- *Et voilà* we have a confidence interval
  - $\hat{\mu}_{\text{low}} = \bar{X} - z_{\alpha_2} \cdot \sigma / \sqrt{n}$
  - $\hat{\mu}_{\text{high}} \bar{X} + z_{\alpha_2} \cdot \sigma / \sqrt{n}$

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Confidence interval for the mean

$$\bar{X} - z_{\alpha_2} \cdot \sigma/\sqrt{n} \leq \mu \leq \bar{X} + z_{\alpha_2} \cdot \sigma/\sqrt{n}$$

- Assumptions:
  1. We know σ
  2. The normal distribution applies, e.g. either
     1. Large n, or
     2. X is normally distributed
- To come:
  - Estimating σ for the normal distribution
  - Handling small samples with Student's t-distribution
  - Estimating the binomial proportion, p, when $X \sim B(n, p)$