Bayes' Law

Calculations with Dependent Events

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Problem

1. Two events A, B
2. Many probabilities of interest
    1. $P(A)$, $P(B)$
    2. $P(A|B)$, $P(B|A)$
    3. $P(A \cap B)$
3. Generally, some are easy to find intuitively
4. Others are difficult
5. Fortunately, they are all linked
6. How do we find one from the others?

Bayes Law

$$P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A|B) \cdot P(B) + P(A|\neg B) \cdot P(\neg B)}$$

aka. Bayes' Equation or Bayes' Rule

Basic Rule #1

Joint Probability

$$P(A \cap B) = P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$$

Basic Rule #2

Decomposing one probability into conditional ones

$$P(A) = P(A|B) \cdot P(B) + P(A|\neg B) \cdot P(\neg B)$$

Bayes Law

- $P(A \cap B) = P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$
- $P(A) = P(A|B) \cdot P(B) + P(A|\neg B) \cdot P(\neg B)$

1. Rearranging and combining the two equations, we get Bayes' equation
2. First, note that:
   - $P(A \cap B) = P(B|A) \cdot [P(A|B) \cdot P(B) + P(A|\neg B) \cdot P(\neg B)]$
   - $P(A \cap B) = P(A|B) \cdot P(B)$
3. Bayes' Rule:

$$P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A|B) \cdot P(B) + P(A|\neg B) \cdot P(\neg B)}$$

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Summary

- Two basic rules about conditional probabilities
  - $P(A \cap B) = P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$
  - $P(A) = P(A|B) \cdot P(B) + P(A|\neg B) \cdot P(\neg B)$
- Bayes' Rule:

$$P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A|B) \cdot P(B) + P(A|\neg B) \cdot P(\neg B)}$$